Indian Statistical Institute, Bangalore

M. Math. First Year

Second Semester - Topology II

Back Paper Exam | Duration: 3 hours | Date : July 11, 2016

Max Marks: 100

Remark: Each question carries 20 marks, split as 5+15 in the two-part questions.
Throughout, X stands for a connected abstract simplicial complex.

1. If (M, x) and (N, y) are two path connected pointed topological spaces then show that $\pi_1(M \times N, (x, y))$ is isomorphic to the direct product of $\pi_1(M, x)$ and $\pi_1(N, y)$.

2. If M is a path connected topological space then show that $\pi_1(M, x)$ and $\pi_1(M, y)$ are isomorphic for any two points x, y of M.

3. Show that $H_1(X)$ is the abelianization of $\pi_1(X)$.

4. (a) If X is d-dimensional then show that $H_d(X)$ is a free abelian group.

   (b) Let X be the simplicial complex whose facets are:

   $$\{1,2,3\}, \{1,2,6\}, \{1,3,4\}, \{1,4,5\}, \{1,5,6\}, \{2,3,5\}, \{2,4,5\}, \{2,4,6\}, \{3,4,6\}, \{3,5,6\}.$$

   Calculate $\chi(X)$ and $H_2(X)$

5. (a) For any vertex x of X, let $X_x = \{\alpha : x \notin \alpha \text{ and } \alpha \cup \{x\} \in X\}$. Then show that $(i+1)f_i(X) = \sum_x f_{i-1}(X_x)$, where the sum is over all vertices of $X, i \geqslant 0$.

   (b) A d-dimensional complex is said to be a pseudo manifold if all its facets are d-dimensional and each $(d-1)-$ face is contained in exactly two facets. Show that any such complex X satisfies $f_i(X) \geqslant \begin{pmatrix} d+2 \\ i+1 \end{pmatrix}$ for all i.